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ROUGHNESS COEFFICIENTS AND VELOCITY ESTIMATION IN WELL-INUNDATED SHEET AND RILLED OVERLAND FLOW WITHOUT STRONGLY ERODING BED FORMS

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ABSTRACT

Even with the flow of water over a soil surface in which roughness elements are well inundated, and in less erosive situations where erosional bed forms are not pronounced, the magnitude of resistance coefficients in equations such as those of Darcy–Weisbach, Chezy or Manning vary with flow velocity (at least). Using both original laboratory and field data, and data from the literature, the paper examines this question of the apparent variation of resistance coefficients in relation to flow velocity, even in the absence of interaction between hydraulics and resulting erosional bed forms. Resistance equations are first assessed as to their ability to describe overland flow velocity when tested against these data sources. The result is that Manning's equation received stronger support than the Darcy–Weisbach or Chezy equations, though all equations were useful.

The second question addressed is how best to estimate velocity of overland flow from measurements of slope and unit discharge, recognizing that the apparent flow velocity variation in resistance coefficients is probably a result of shortcomings in all of the listed resistance equations. A new methodology is illustrated which gives good agreement between estimated and measured flow velocity for both well-inundated sheet and rill flow. Comments are given on the predictive use of this methodology. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS shallow flow hydraulics; roughness coefficients; flow velocity; Manning's equation; Darcy–Weisbach; sheet flow; rill flow

INTRODUCTION

When overland flow of water occurs in actively eroding rills, Govers (1992) and Nearing *et al.* (1997) have shown that the resistance offered to such flows is dominated by the bed forms generated by the erosion processes. In such actively eroding rills, these authors have shown that the classical form of decrease in the Darcy–Weisbach friction factor, f, with flow Reynolds number (Re) (Yoon and Wenzel, 1971) does not apply, and there would be similar limitations with alternative measures of hydraulic resistance.

In natural environments, overland flow can be sufficiently shallow that it flows around roughness elements which are not completely inundated by the flow. In such circumstances, Lawrence (1997) has shown that the degree to which the roughness elements are inundated plays a dominant role in determining the resistance offered to the flow by the surface. Using a wide range of data sources, Lawrence (1997) found that the frictional resistance could be approximately related to an 'inundation ratio' defined as the ratio of flow depth to a scale of the surface roughness features.

Shallow flow over a stone-covered desert surface provides a particular example of flow that may not completely inundate all roughness elements. In this context, Abrahams and Parsons (1994) conceptually partitioned the resistance to overland flow into the following forms: grain resistance of the surface between the stones; form resistance (exerted by stones), wave resistance generated by disturbance of the free water surface due to flow over and around stones; and, finally, rain resistance due to impact of rainfall. Abrahams and Parsons (1994) showed that for Froude numbers (Fr) greater than about 0·5, wave

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Table I. Some physiochemical characteristics of soil at the field site (Goomboorian loamy sand), the soil also used in the laboratory experiments

Depth (m)	pH* (1:5)	EC (dS m^{-1})	OM† (%)	Coarse sand (%)	Fine sand (%)	Silt (%)	Clay (%)
0–0·10	6	0·17	1·2	40	53	5	2

* Aqueous 1:5, soil : water.
† OM = organic matter.

resistance can be the dominant resistance to flow if stone cover is greater than 10 per cent. Flow in rills is commonly accompanied by some disturbance of the free water surface, and so this finding of Abrahams and Parsons (1997) suggests that frictional resistance in rill flow might depend on the Froude number.

This paper returns to consider some of the questions which remain, even when surface roughness elements are well inundated, and where there is not the strong interaction between hydraulics and erosional bed forms investigated by Nearing *et al.* (1997), for example. This paper gives particular attention to the velocity of overland flow. This is because this flow velocity is involved in the theory of a number of new-generation, physically based soil erosion models such as WEPP (Laflen *et al.*, 1991), EUROSEM (Morgan *et al.*, 1992) and GUEST (Rose, 1993; Misra and Rose, 1996; Rose *et al.*, 1997).

The two questions considered in this paper are as follows.

1. What ranking does experimental data give to the three long-established, analytically interrelated but different resistance equations associated with the names of Manning, Darcy, Weisbach and Chezy? In particular, how well do they serve as descriptors of overland flow velocity when tested against experimental data?
2. How can flow velocity best be estimated? A commonly employed method to obtain velocity is to appeal to an experimentally determined relationship for a friction factor, such as the commonly observed decrease in the Darcy–Weisbach friction factor, f, with flow Reynolds number, Re (Chow, 1959; Savat, 1980). However, Emmett (1970) and the work reported earlier have drawn attention to limitations in this approach. This paper develops and tests an alternative procedure.

In considering these two questions, the analysis covers both original data by the authors, reported in this paper, as well as appropriate data from the literature. Anticipating the outcome of the investigation of appropriate forms of resistance equation to use, the experimental methodology employed will be described in terms of determining Manning's n.

EXPERIMENTS

Laboratory experiments

Laboratory experiments were undertaken in the 5·8 m by 1·0 m flume of the Griffith University Tilting-Flume Simulated Rainfall facility (or GUTSR) described by Misra and Rose (1995). In this facility, rainfall was simulated using a series of spray nozzles supported 9 m above the flume by a metal frame. A rainfall rate of approximately 100 m h^{-1} was used in these experiments. The soil used in this laboratory study was a Goomboorian loamy sand, an Albic Arenosol (FAO–Unesco, 1990) from the site of the field experiment (described later). Results of particle-size distribution analysis using the hydrometer method and some physiochemical characteristics of this soil are given in Table I.

Three types of experiments were carried out: run-on alone, rainfall alone, rainfall plus run-on. In experiments with run-on alone and rainfall plus run-on, clear water was added from the top of the flume by a self-levelling, constant-head device connected to the water supply. A series of volumetric fluxes ranging from 0·162 to 0·670 l s^{-1} were used as run-on. The slope of the 1 m wide flume was adjusted,

Table II. Range of slope and unit discharge investigated in the laboratory and field experiments reported in this paper

Data source	Sheet/rill flow	Number of experiments	Slope range (%)	Unit discharge range (10^{-3} m^2 s^{-1})
Laboratory experiments	Sheet	51	0·05–3·50	0·16–0·67
Rouhipour (1997)	Rill*	23	0·50–10·50	1·03–2·39
Field (Goomboorian) (C.A.A. Ciesiolka, pers. comm.)	Furrow	19	3·20–5·00	0·36–8·21

* A single rill was pre-formed in the flume's soil bed.

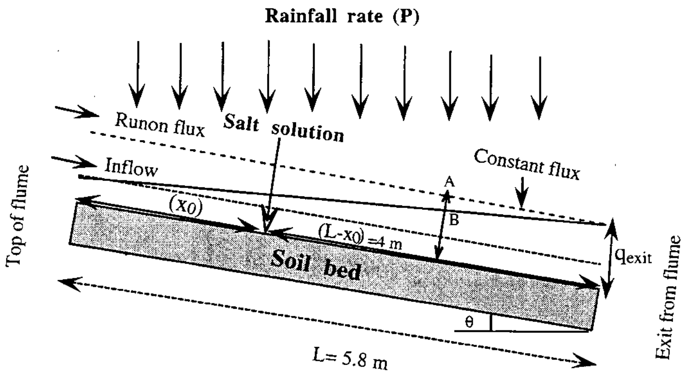

Figure 1. Schematic essentials of the laboratory equipments of Rouhipour (1997). Flow alone over the saturated soil bed, in which inflow and outflow are equal, has a water surface represented by the dashed lines. When rainfall of constant rate is added, the solid line indicates the increase in flux down the flume. The location of salt solution addition for the salt-tracing measurement of flow velocity is also indicated

ranging from 0·1 to 3·5 per cent for sheet flow, and 0·5 to 10·5 per cent for experiments with a single rectangular rill (described later). Details of various combinations of fluxes and slopes are given in Table II.

In experimental conditions where prior experiments showed that rills would form, the inaccuracies that would be introduced by seeking to make measurements on braided rills were minimized by ensuring that only a single rill formed. This was achieved by partial pre-forming of the rill, and allowing natural erosion processes at the particular slope and flow rate to generate the single rill then used to investigate the hydraulics of flow in pre-formed rills. The types of rills for which results are reported were formed at a stage where erosion was not so great as to produce visible bed forms. In the loamy sand used in these experiments, rills formed with a close-to-rectangular geometry, and rill-width measurement at 1 m intervals showed that width increased somewhat with downslope distance as well as with time due to erosion of the rill wall.

The velocity of steady overland flow was determined using the salt-tracing technique (Luk and Merz, 1992). Because the sandy loam had little structure, sodium chloride could be used as the conductivity trace, the output of an electrical conductivity meter placed at exit from the flume being electronically recorded. As shown in Figure 1, a pulse of salt solution was introduced at distance $x_0 = 1{\cdot}8$ m from the top of the flume of length L ($=5{\cdot}8$ m), so that $(L-x_0) = 4$ m. The mean flow velocity was calculated from the mean time of travel in peak conductivity, which was well defined. The plot of conductivity versus time was close to symmetrical in most experiments, which were replicated three times. The maximum difference in arrival time between the peak and centroid of electrical conductivity was 13 per cent, but was mostly much smaller.

Table III. Details of experiments from the literature employed in testing the three resistance equations

Data source	flow type	Number of experiments	Slope range (%)	Range of unit discharge (10^{-3} m^2 s^{-1})
Guy *et al.* (1990)	Sheet	20	2·0–20	0·01–0·24
Li and Abrahams (1997)	Sheet	105	2·0–9·5	0·17–3·67

Field experiments

Field experiments were located on a pineapple farm at Goomboorian, southeast Queensland, Australia (26°36′S, 153°18′E). Description of the loamy sand soil was given in Table I. Soil loss from pineapple farming on this erodible soil is reduced by orienting the planting beds and associated furrows partly across slope at less than 6 per cent. Flow resistance experiments were carried out in furrows of width from 0·13 to 0·33m formed between the raised planting beds. Water was introduced to the top end of 7 to 8m long furrows for a sufficient time until steady flow conditions were achieved. Flow rate was measured by calibrated flow meters. Velocity of flow was assessed using an introduced dye. The velocity of the leading edge of the dye was multiplied by a reduction factor to give an approximate estimate of mean velocity. The value of two-thirds was chosen for this reduction factor (Li and Abrahams, 1997).

Experiments in the literature

In addition to the author's experiments listed in Table II, two sets of data from the literature were also used to test the relative applicability of alternative resistance equations in the absence of strongly eroding bed forms. Details of the literature data sets employed are given in Table III. Both experiments were carried out on a planar sandy bed, so the grain roughness would be the dominant roughness form. The experiments of Guy *et al.* (1990) had sediment in the flowing water, but the experiments of Li and Abrahams (1997) were free of sediment.

Determination of resistance coefficients with sheet flow. Since the flume had an impermeable base and the soil bed was saturated prior to each experiment, the run-off rate per unit area due to rainfall is equal to the (constant) rainfall rate applied (P). Thus the volumetric flux of water per unit width, q, increases with distance down the flume, x, as:

$$q(x) = q_{\text{in}} + Px \tag{1}$$

where q_{in} is the value of q at entry to the flume.

Whilst flow velocity increases with x, the mean velocity of flow, $\bar{V}$, along the measurement distance from salt input (L–x_0) (see Figure 1) is given by:

$$\bar{V} = (L - x_0)/t \tag{2}$$

where t is the mean travel time of salt solution given by:

$$t = \int_{x_0}^{L} \frac{\mathrm{d}x}{V} \tag{3}$$

The velocity, V, can be described, for example, by Manning's equation for sheet flow:

$$V = S^{1/2}\, D^{2/3}/n \tag{4}$$

where S is the slope (sin θ, Figure 1), D is the depth of flow, and n is Manning's friction coefficient.

Substituting for D from Equation 4 to eliminate D from $q = DV$, it follows from Equation 1 that:

$$V = (S^{1/2}/n)^{3/5}\ (q_{\rm in} + Px)^{2/5} \tag{5}$$

Substituting for V from Equation 5 into Equation 3 gives an expression for t, and substituting this expression into Equation 2 yields the following expression for Manning's n:

$$n = \left(3/5\right)^{5/3}\ S^{1/2}\ \left(\frac{P}{\overline{V}}\right)^{5/3}\ \frac{(L - x_0)^{5/3}}{\left[\left(q_{\rm in} + PL\right)^{3/5} - \left(q_{\rm in} + Px_0\right)^{3/5}\right]^{5/3}} \tag{6}$$

Equation 6 was used to calculate n in experiments with rainfall alone or rainfall plus run-on. With run-on alone, Manning's n was calculated from:

$$n = q^{2/3}\ S^{1/2}/V^{5/3} \tag{7}$$

q, S and V being the directly measured quantities. Similar relations were developed for Darcy–Weisbach f and Chezy C.

Determination of resistance coefficients with flow in a single rill. Experiments in the flume with a single pre-formed rill were carried out with run-on alone (without rainfall). The flow velocity, V, was again spatially variable due to gradual increase in the measured width, W, of the rill with distance down the flume. Thus, again using Manning's Equation:

$$V = (S^{1/2}/n)R_{(x)}^{2/3} \tag{8}$$

where $R_{(x)}$, the hydraulic radius, is also spatially variable. Thus the average velocity of flow from salt injection (at $x = x_0$) to the end of the flume, $\bar{V}$, is given by:

$$\begin{aligned} \bar{V} &= \int_{x_0}^{L} V\mathrm{d}x/(L - x_0) \\ &= \frac{S^{1/2}}{n(L - x_0)} \int_{x_0}^{L} R_{(x)}^{2/3}\ \mathrm{d}x \end{aligned} \tag{9}$$

Manning's n was calculated from Equation 9 with the integral being evaluated numerically.

It follows from Equation 9 that an effective average hydraulic radius, $R_{\rm av}$, can be defined for Manning's equation by:

$$R_{\rm av}^{2/3} = \int_{x_0}^{L} R_{(x)}^{2/3}\ \mathrm{d}x/(L - x_0) \tag{10}$$

A similar procedure was followed for the other resistance equations.

RESULTS AND DISCUSSION

Comparison of Manning, Darcy–Weisbach and Chezy flow-resistance equations

Despite the advantage of the Darcy–Weisback f being non-dimensional, Manning's n and Chezy C are still commonly used. These three resistance equations will now be compared in terms of their ability to

Table IV. The expressions for Manning's n, Darcy–Weisbach's f and Chezy's C for sheet and rill flow in terms of their definition, and in terms of directly measured variables given in the text. Used for flow without added rainfall

Equation	Sheet/Rill	Definition	Expression for V in terms of measured variables
Manning's n	Sheet	$n = \frac{S^{1/2}\ D^{2/3}}{V}$	$V = \frac{q^{0\cdot4}\ S^{0\cdot3}}{n^{0\cdot6}}$
	Rill	$n = \frac{S^{1/2}\ R^{2/3}}{V}$	$V = \frac{S^{0\cdot3}}{n^{0\cdot6}}\left(\frac{G}{W+2D}\right)^{0\cdot4}$
Darcy–Weisbach's f	Sheet	$f = \frac{8g\ DS}{V^2}$	$V = (8gqS/f)^{0\cdot333}$
	Rill	$f = \frac{8g\ RS}{V^2}$	$V = \left[\frac{8g}{f}\left(\frac{SG}{W+2D}\right)\right]^{0\cdot333}$
Chezy's C	Sheet	$C = \frac{V}{(DS)^{1/2}}$	$V = C^{0\cdot667}\ (qS)^{0\cdot333}$
	Rill	$C = \frac{V}{(RS)^{1/2}}$	$V = C^{0\cdot667}\left(\frac{SG}{W+2D}\right)^{0\cdot333}$

describe the five data sets listed in Tables II and III.

Table IV gives the definition of the three resistance coefficients, both for sheet flow and for flow in a rectangular rill. Also given is the corresponding expression for the velocity of flow, V, in terms of the commonly measured quantities which for sheet flow are the slope, S, and volumetric flux per unit width of flow or unit discharge, q ($m^2\ s^{-1}$). For flow in a rectangular rill of width W and hydraulic radius R, q (used in Table II) is replaced by Table IV by the volumetric flux G ($m^3\ s^{-1}$). The depth of water, D, used in the expression for V in rills in Table IV, was calculated from the measured variables using the expression $D = G/WV$.

In the sheet-flow experiments of Rouhipour (1997) in which rainfall contributed to make flow spatially variable, more complex equations than those given in Table IV were used, such as is illustrated for Manning's equation by Equation 6.

The methodology adopted to pursue the first question was to plot the experimentally measured value of V in each data set against the combination of measured variables S, q (or G, W and the calculated water depth, D) which are on the right-hand side of the expressions for V in Table IV. Since the resistance coefficients n, f and C are conceptually constant in any experiment, and the acceleration due to gravity, g is also constant, V was plotted against the appropriate combination of measured quantities given by the expressions for V.

It follows from the formulae in Table IV that the dependence of V in terms of other measured variables S, q (or G, W and D) is identical for the Darcy–Weisbach and Chezy equations. Thus, in this method of equation testing, only one equation of these two needs to be tested, and Darcy–Weisbach was used.

The method of testing the Manning and Darcy–Weisbach equations is illustrated in Figure 2 for the latter equation using the experimental data of Li and Abrahams (1997). If the tested equation was a completely adequate description of error-free data, then the relationship fitted to the data should be linear with no scatter, and pass through the origin with no intercept. Neither of the equations tested met

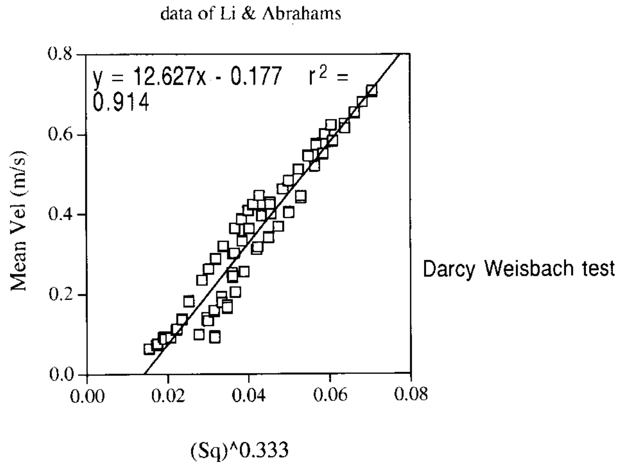

Figure 2. A test of the ability of the Darcy–Weisbach equation to describe the sheet-flow data of Li and Abrahams (1997). From the expression for V in Table IV, the relationship should be linear and pass through the origin of the plot (□, mean velocity, $\mathrm{m\,s^{-1}}$)

Table V. The result of testing each of the two resistance equations for sheet-flow data for each of the data sources listed in Tables II and III

Resistance equation	Data source (N = number of experiments)					
	Li and Abrahams (1997) (N = 105)		Rouhipour (1997) (N = 51)		Guy *et al.* (1990) (N = 20)	
	r^2	Intercept fraction	r^2	Intercept fraction	r^2	Intercept fraction
Manning	0·961	0·180	0·889	0·176	0·954	0·120
Darcy–Weisbach	0·914	0·212	0·818	0·147	0·908	0·145

this ideal requirement for any of the data sets listed in Tables II and III, as is illustrated in Figure 2.

Two quantitative measures were used to test the adequacy of the resistance equations. The first was the value of r^2 given by a linear fit to the data of the type illustrated in Figure 2. The second test is the magnitude of the intercept on the y-axis representing V, expressed as the ratio of the magnitude of this intercept ($|b|$), divided by the sum of $|b|$ and the maximum value of V on the y-axis. This ratio is called the 'intercept fraction'. The closer the value of r^2 to unity, and the smaller the intercept fraction, the stronger the support for the ability of the particular resistance equation to describe the measured flow velocity in terms of other measured quantities.

Table V gives the results of these two tests for sheet-flow data listed in Tables II and III. The results in Table V show that for all three data sets, the value of r^2 was higher for Manning's equation than for Darcy–Weisbach's (and thus for Chezy also). In two data sets, the intercept fraction was lower for Manning's than for Darcy–Weisbach's equation. Thus, in this type of test, Manning's equation received stronger support than the two alternatives. Manning's equation is often considered to be applicable only to turbulent flow, however, flow would not be turbulent in the majority of the sheet-flow data considered, especially at lower slopes and fluxes. Rainfall in some of the experiments of Rouhipour (1997) would have added some turbulence, but this did not noticeably improve the performance of Manning's equation.

Table VI gives the results of the same type of resistance equation testing for all the rill data listed in Table II. The rills did not have extensive bed forms.

Table VI. The results of testing each of the two resistance equations for rill flow data from the data sources listed in Table II

Resistance equation	Data source (N = number of experiments)			
	Rouhipour (1997) (laboratory) ($N = 26$)		Field (Goomboorian) ($N = 19$)	
	r^2	Intercept fraction	r^2	Intercept fraction
Manning	0·872	0·206	0·928	0·085
Darcy–Weisbach	0·832	0·165	0·921	0·162

The results in Table VI show that, as for the sheet-flow results in Table V, Manning's equation again gave the higher value of r^2. The intercept fraction was higher for Manning's equation in the experiments of Rouhipour (1997), and higher for the Darcy–Weisbach equation for the field data.

In conclusion, none of the resistance equations gave a perfect representation of the experimental data against which they were tested, but support for Manning's equation was generally stronger than for the Darcy–Weisbach (and hence Chezy) equations. Thus, in the following subsection, Manning's equation will be used to illustrate a method of estimating the velocity of overland flow which in particular deals with the problem introduced by a non-zero intercept. This problem is that the value of any of the resistance coefficients will vary with flow velocity (at least) for flow over a given surface, even when roughness elements are well inundated, and in the absence of erosional bed forms.

A method for estimating the velocity of overland flow

This subsection develops and illustrates a methodology for estimating velocity of overland flow of water from measurements of slope (S, the sine of the slope angle) and discharge per unit width of water (unit discharge, q). The data sources used earlier in the paper, in which flow velocity was also measured, were used to develop the methodology. Comments on predictive use of the methodology will then be given.

First, a general equation will be developed for the velocity of flow, V (m s^{-1}), in both sheet flow and flow in a rectangular rill of width W. For either geometries of flow, using Manning's equation:

$$V = \frac{S^{1/2}}{n} R^{2/3} \tag{11}$$

where, in sheet flow, the hydraulic radius, R, is equal to water depth, D. In either sheet flow or flow in a rill, the unit discharge (m^2 s^{-1}) is given by:

$$q = DV \tag{12}$$

Substituting $R = WD/(W + 2D)$ into Equation 11 and using Equation 12, it follows that:

$$V = \frac{S^{0\cdot3}}{n^{0\cdot6}} \left(\frac{q}{1 + \eta} \right)^{0\cdot4} \tag{13}$$

where parameter η is related to the width-to-depth ratio:

$$\eta = 2D/W \tag{14}$$

For sheet flow, $\eta = 0$ can be assumed.

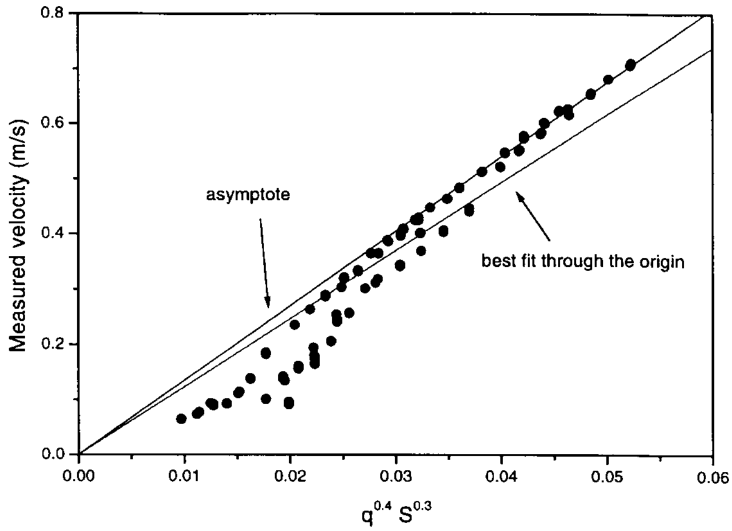

Figure 3. A plot of measured mean velocity V from the data of Li and Abrahams (1997) against the quantity of $q^{0.4}S^{0.3}$ to which it should be simply linearly related if Manning's equation provided a perfect description of sheet flow. The slope of the best fit through the origin of the plot can be regarded as a basic model from which to estimate velocity

Table VII. Summary of the adequacy of three different models to fit the data of Li and Abrahams (1997) shown in Figure 3

Model	Parameters	RMSE ($m\,s^{-1}$) (% of mean)	Performance
Straight line	$A=-0{\cdot}146$**, $B=16{\cdot}8$	0·035 (9·7)	$r^2=0{\cdot}96$
Through the origin	$B=12{\cdot}31$ ($n=0{\cdot}0152$)	0·061 (16·9)	$r^2=0{\cdot}96$
Equation 16 (two-step method)	$n_0=0{\cdot}0131$, $Re_0=1827$	0·01515 (4·5)	$E=0{\cdot}99$
Equation 16 (downhill simplex)	$n_0=0{\cdot}0130$, $Re_0=1851$	0·01512 (4·4)	$E=0{\cdot}99$

** Significantly different from zero (p value $<0{\cdot}0001$) value of intercept
B is defined in Equation 15

Should Manning's equation (and hence Equation 13) provide a perfect description of sheet flow ($\eta=0$), then a plot of V against $S^{0.3}\,q^{0.4}$ should be fitted by a straight line through the origin of slope B where:

$$n = B^{-5/3} \tag{15}$$

Figure 3 shows such a plot for the data of Li and Abrahams (1997). This figure illustrates a feature typical of all data examined by the authors: the slope B of a line from the origin to points in Figure 3 at lower values of velocity or q is lower, and hence n high (Equation 15), than at higher velocities or discharges. If the nature of the flow surface is essentially the same regardless of V or q (as is the case for these data), then n should remain constant. That n increases at lower values of V or q is probably an indication that a partial violation of Manning's equation increases as V decreases to low values. This limitation is also shared by the Chezy and Darcy–Weisbach equations.

Fitting the data in Figure 3 with an unconstrained linear relation gives slope $B=16{\cdot}8$ and an intercept on the V-axis significantly different from zero (Table VII). A similar linear fit to the data which is forced to go through the origin gives $B=12{\cdot}31$ (Table VII), so that $n=0{\cdot}0152\,m^{-1/3}s$ (Equation 15). This value

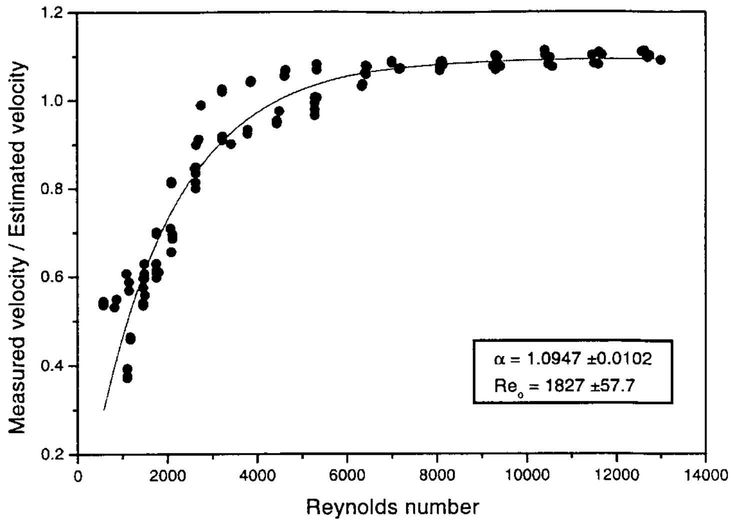

Figure 4. The ratio of experimentally measured velocity shown in Figure 3 to the velocity estimated using the linear best fit through the origin in Figure 3, shown as a function of the Reynolds number of the flow and fitted by a relationship presented in the text

of n can be regarded as a first-order approximation. Table VII also shows the root mean squared error (RMSE), the RMSE as a percentage error of the mean, and r^2 for these fits to the data.

The linear fit to the data through the origin can be regarded as the basic model. Figure 3 shows that this basic model overestimates the velocity when the velocity is low, and underestimates when the velocity is high, although this underestimation is not as serious in percentage error terms as the overestimation at lower velocities. Thus there is a consistency in the pattern of residuals from the basic model, and similar consistencies in residual patterns have been found for similar plots with other data sources, and from comparable plots for the other two resistance equations (cf. the previous subsection of this paper).

How this pattern of residuals might be related to other measured parameters was investigated. The most coherent relationship found is that illustrated in Figure 4 for the data set of Li and Abrahams (1997). The ratio of measured velocity to that estimated using the basic model is shown to be coherently related to the flow Reynolds number. This pattern of discrepancy or residuals was well fitted by an exponential function of the form $\alpha\,[1-\exp\,(-Re/Re_0)]$, where, as shown in Figure 4, α is close to unity.

Hence Equation 13 was modified to the following form:

$$V = \frac{[1-\exp(-Re/Re_0)]}{n_0^{0\cdot6}}\left(\frac{q}{1+\eta}\right)^{0\cdot4} S^{0\cdot3} \tag{16}$$

where n_0 may be called a baseline value of Manning's n which is asymptotically approached at large Reynolds numbers, and Re_0 is a characteristic Reynolds number, which it will be shown can be broadly related to the transition from laminar to turbulent flow.

The modified model given by Equation 16 was fitted to the data of Li and Abrahams (1997), shown in Figures 3 and 4 using a first-order approximation and residual analysis, referred to as the two-step method in Table VII. The parameters n_0 and Re_0 were also optimized using the downhill simplex method (Nelder and Mead, 1965; Press *et al.*, 1992), with results only marginally better than the two-step method (Table VII). The term E in Table VII given for these two parameter-optimizing methods is the measure of model performance efficiency given by Nash and Sutcliff (1970), $E = 1{\cdot}0$ indicating perfect agreement between model and data.

Table VIII. Summary model results for five sources of data described in the text

Model	Variable	Guy (*et* al. (1990)	Li and Abrahams (1997)	Rouhipour (1997) Sheet	Rouhipour (1997) Rill	Field (Goomboorian) (Ciesiolka, pers. comm.)
	No. of data points	20	105	51	23	19
	Re-range	11–239	583–13 000	648–2680	3789–9541	367–6975
	Average V ($m\,s^{-1}$)	0·138	0·340	0·094	0·193	0·288
Equation (16)	n_0	0·0095	0·0130	0·0140	0·0225	0·0257
	Re_0	141	1851	794	2857	1360
	E	0·97	0·99	0·92	0·83	0·92
	RMSE ($m\,s^{-1}$)	0·021	0·015	0·016	0·037	0·037
	(% of mean)	(14·9)	(4·4)	(16·5)	(18·9)	(12·8)
Straight line through origin	n	0·0110	0·0152	0·0189	0·0282	0·0263
	RMSE ($m\,s^{-1}$)	0·035	0·061	0·024	0·040	0·038
	(% of mean)	(25·3)	(16·9)	(26·5)	(21·0)	(13·3)

Using the modified model in Equation 16 and the simplex downhill method for optimizing parameters n_0 and Re_0, Table VIII summarizes results obtained for all four sources of data described earlier in this paper.

From the estimated values of Re_0 shown in Table VIII, it can be seen that Re_0 appears to be constrained by the Reynolds number range covered in the experiment. However, the maximum value of Re_0 is less than 3000, even in the rilled data of Rouhipour (1997), where Re was up to 9541. At the higher Reynolds number characteristic of the rilled experiments of Rouhipour (1997), flow is likely to be fully turbulent; and, as shown in Table VIII, introduction of the modification involving the exponential term in Re was least effective in reducing the RMSE compared to the basic straight line through the origin model.

For sheet flow, the value of n_0 obtained from the three sources of data shown in Table VIII is reasonably consistent, the value of n_0 with a rectangular rill being between two and three times higher.

The parameters in Table VIII, estimated by fitting the modified model of Equation 16 to the five data sets, were then used to estimate velocity of flow. Figure 5 compares these estimated with measured velocities, indicating the adequacy of Equation 16 as a model, provided parameters n_0 and Re_0 are determined.

DISCUSSION AND CONCLUSION

The data used in this paper all fitted within the adopted experimental limitations, namely complete inundation of roughness elements, and sufficiently modest erosional conditions that pronounced irregular erosional bed forms were not produced. Mean velocity of flow was measured in all data using salt-tracing except for that of Guy *et al.* (1990) and Ciesiolka (pers. comm.), in which case mean velocity was estimated by multiplying dye-measured surface velocity by a correction factor of two-thirds. As shown by Li and Abrahams (1997), the value of this correction factor can vary in different experimental conditions, there being a dependence on Reynolds number for sediment-free flows on a fixed bed.

The data used from Li and Abrahams (1997) and Guy *et al.* (1990) were determined using a fixed, sand-covered bed, the former being the only data from sediment-free flow. Similar values of n_0 (Table VIII) for all three sets of sheet flow data and the Goomboorian field data of Ciesiolka (pers. comm.) would appear to indicate that the presence of sediment at the modest concentrations in these non-severely eroding situations had only limited effect on this baseline value of Manning's n. The sheet-flow

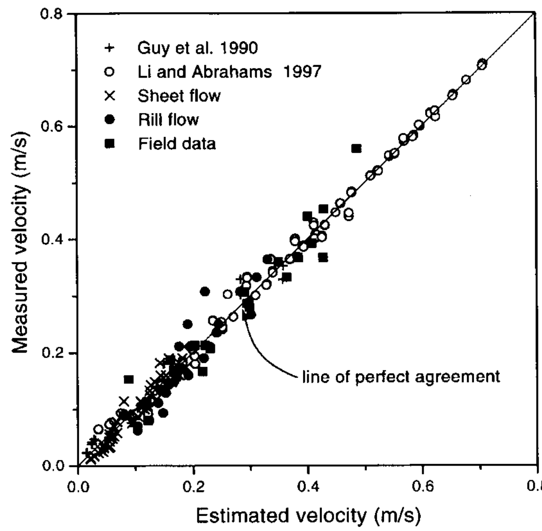

Figure 5. Measured flow velocity versus velocity estimated using the modified flow-velocity model given in Equation 16. The data presented are those of Guy *et al.* (1990), Li and Abrahams (1997), the sheet flow and rill data of Rouhipour (1997), together with the field data for the Goomboorian site (Ciesiolka, pers. comm.) reported in this paper

data of Rouhipour (1997) incuded experiments with and without a rainfall of rate 100mm h^{-1}. Over the range of experimental conditions investigated, given in Table II, this rainfall did not significantly modify the value of calculated resistance coefficients. The value of n_0 was significantly increased in the well incised, pre-formed rills in the experiments of Rouhipour (1997). It is interesting that flow in the pineapple bed furrows in field experiments of Ciesiolka (pers. comm.), the soil being sandy, yielded values of n_0 similar to those measured in rill flow (Table VIII). Flow width in the furrows varied from 0·13 to 0·33m.

The data analysis summarized in Tables V and VI indicates that all three resistance equations investigated provided a good coherent basis for summarizing the experimental data, Manning's equation consistently giving the highest values of r^2. However, except for the data of Guy *et al.* (1990), all three resistance equations exhibited non-zero intercept fractions, and these were highest for Manning's equation. The implication of this non-zero intercept fraction is that there is an apparent variation with flow in each of the resistance coefficients. The nature of this variation is a decline in magnitude of the resistance coefficients, Manning's n and Darcy–Weisbach f is flow velocity (or flow velocity-related factors) increase, the reverse being the case for Chezy C because of its inverse definition (Table IV). In classic fluid mechanics, the decline in drag coefficients with increasing Reynolds number prior to achieving fully turbulent flow is well known (Chow, 1959). Since f (and so n) is related to a drag coefficient, it is possible that the dependence of the resistance coefficients (as defined) on flow velocity is real. However, since roughness elements were completely inundated in all experiments whose data were interpreted, it is also possible that this flow dependence of resistance coefficients is more apparent than real, being a consequence of limitations in each of the three resistance equations investigated.

From Equation 11, an inverse dependence of Manning's n on flow velocity V is expected, the dependence on S and R (or D) being less pronounced. For the 51 flow alone and flow plus rainfall sheet-flow laboratory experiments of Rouhipour (1997) given in Table II, it can be shown that the apparent flow dependence of n is well represented by:

$$N = 0{\cdot}005\ V^{-0{\cdot}65}\ (r^2 = 0{\cdot}80) \tag{17}$$

The discrepancy between the exponent on V in Equation 17 and the theoretically expected value of –1 is presumably due both to inadequacy in Manning's equation noted earlier and the summary of data

over a range of slopes and flow depths. Using Equation 17 together with Equation 13 for sheet flow (i.e. $\eta = 0$) gives values of flow velocity in approximate agreement with the results of Nearing *et al.* (1997) reported as in the 'laminar flow regime'.

However, the model modification given in Equation 16 is a better way of dealing with this problem of apparent flow dependence, in which only one parameter (Re_0) is introduced. In predictive use of this model, where only q and S are known, n_0 and Re_0 would still need to be chosen from relevant experience. Based on the experience reported in this paper, the value of Re_0 should be within the range of transitional flow (laminar to turbulent), but if (as in the data of Guy *et al.* (1990)) the range of Re is restricted, a choice of Re_0 within the experimental range of Re should lead to acceptable error. From Table VIII, the value of n_0 for bare sandy soil would appear to be approximately $0{\cdot}01\,\mathrm{m}^{-1/3}\,\mathrm{s}$, provided pronounced rilling has not developed, in which case Manning's n_0 could be increased by a factor of two or more (Table VIII).

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REFERENCES

Abrahams, A. D. and Parons, A. J. 1994. 'Hydraulics of interrill overland flow on stone-covered desert surfaces', *Catena,* **23**, 111–140.

Chow, V. T. 1959. *Open Channel Hydraulics*, McGraw-Hill, New York.

Emmett, W. W. 1970. *The hydraulics of overland flow on hillslopes*, US Geological Survey Professional Paper **662-A**, 68 pp.

FAO–Unesco. 1990. *Soil Map of the World: Revised Legend*, World Soil Resources Report **60**, FAO, Rome.

Govers, G. 1992. 'Relationships between discharge, velocity and flow area for rills eroding loose, non-layered materials', *Earth Surface Processes and Landforms,* **17**, 515–528.

Guy, B. T., Dickinson, W. T., Rudra, R. P. and Wall, G. 1990. 'Hydraulics of sediment-laden sheetflow and the influence of simulated rainfall', *Earth Surface Processes and Landforms,* **15**, 101–118.

Laflen, J. M., Lane, L. J. and Foster, G. R. 1991. 'WEPP: A new generation of erosion prediction technology', *Journal of Soil and Water Conservation,* **46**, 34–38.

Lawrence, D. S. L. 1997. 'Macroscale surface roughness and frictional resistance in overland flow', *Earth Surface Processes and Landforms,* **22**, 365–382.

Li, G. and Abrahams, A. D. 1997. 'Effect of saltating sediment load on the determination of the mean velocity of overland flow', *Water Resources Research,* **33**, 341–347.

Luk, S. H. and Merz, W. 1992. 'Use of the salt tracing technique to determine the velocity of overland flow', *Soil Technology,* **5**, 289–301.

Misra, R. K. and Rose, C. W. 1995. 'An examination of the relationship between erodibility parameters and soil strength', *Australian Journal of Soil Research,* **33**, 715–732.

Misra, R. K. and Rose, C. W. 1996. 'Application and sensitivity analysis of process-based erosion model GUEST', *European Journal of Soil Science,* **47**, 593–604.

Morgan, R. P. C., Quenton, J. N. and Rickson, R. J. 1992. *EUROSEM: Documentation Manual*, Silsoe College, Solsoe, UK.

Nash, J. E. and Sutcliffe, J. V. 1970. 'River flow forecasting through conceptual models, Part 1: A discussion principles', *Journal of Hydrology,* **10**, 282–290.

Nearing, M. A., Norton, L. D., Bulgakov, D. A., Larionov, G. A., West, L. T. and Dontsova, K. M. 1997. 'Hydraulics and erosion in eroding rills', *Water Resources Research,* **33**, 865–876.

Nelder, J. A. and Mead, R. 1965. 'A simplex method for function minimization', *Computer Journal,* **7**, 308–313.

Press, W. H., Teukolsky, S. A., Vetterling, W. T. and Flannery, B. P. 1992. *Numerical Recipes*, Cambridge University Press, Cambridge.

Rose, C. W. 1993. 'Erosion and sedimentation', in Bonnell, M., Hufschmidt, M. M. and Gladwell, J. S. (Eds), *Hydrology and Water Management in the Humid Tropics*, Cambridge University Press, Cambridge, 301–343.

Rose, C. W., Coughlan, K. J., Ciesiolka, C. A. A. and Fentie, B. 1997. 'Programe GUEST (Griffith University Erosion System Template)', in Coughlan, K. J. and Rose, C. W. (Eds), *A New Soil Conservation Methodology and Application to Cropping Systems in Tropical Steeplands*, ACIAR Technical Report **40**, 35–58.

Rouhipour, H. 1997. *Interaction between flow-driven and rainfall-driven soil erosion processes*, unpublished PhD thesis, Griffith University, Queensland, Australia.

Savat, J. 1980. 'Resistance to flow in rough supercritical sheet flow', *Earth Surface Processes,* **5**, 103–122.

Yoon, Y. N. and Wenzel, H. G. 1971. 'Mechanics of sheet flow under simulated rainfall', *Journal of the Hydraulics Division, Proceeding of the American Society of Civil Engineers,* **97**, HY9, 1367–1386.

